# Predicting bulk permeability using outcrop fracture attributes: The benifits of a Maximum Likelihood Estimator

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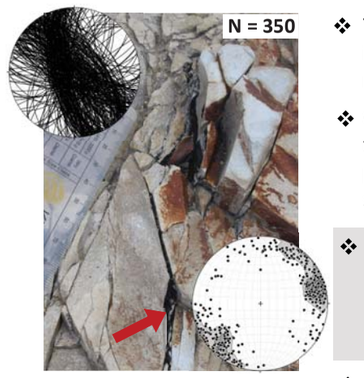
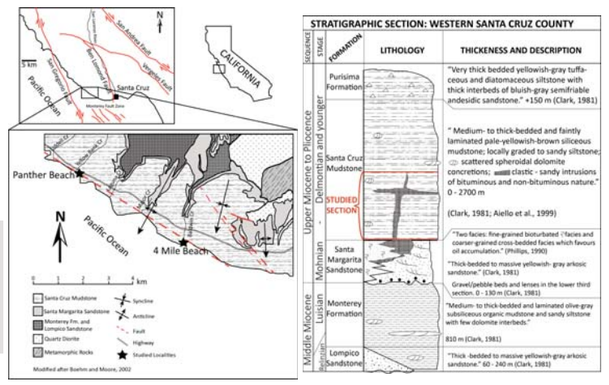
## 1. Summary and Geological Overview




Close caption of bitumen filling a fracture (red arrow) in locality Panther Beach. The fracture systems form a joint set striking predominantly NW (stereoplots).

- The structural arrangement of fracture netwoks plays an important role on the effective flow responce, especially for rocks with low primary porosity and permeability.
- Optimal characterisation of fracture attributes is fundamental to estimate bulk permeability and fluid flow. In order for these parameters to be consistent at small and large scales, the statistical methods used to estimate them play a crucial role.
- Using data collected exclusively on outcrops, the **main objective of this work** is to create a **simple** and **accurate** toll for building **quantitative permeability models**, which can be easily **upscaled to larger areas**.
- The study area is a highly fractured upper Miocene biosiliceous mudstone formation in Santa Cruz (California, USA). These outcrops expose a recently active bitumen-bearing field, which represents a geological analogue of a fractured top seal.

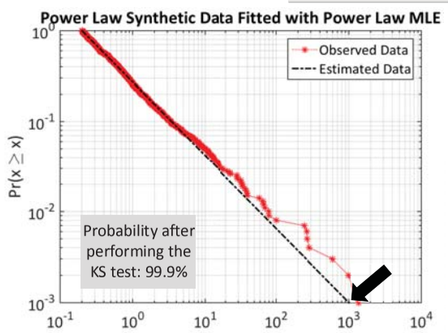



Regional Geology of the north Santa Cruz county (right). Regional stratigraphy of the north Santa Cruz County. Framed in the red box is the stratigraphic interval exposed in the studied areas. (modified after *Boehm & Moore*, 2002).

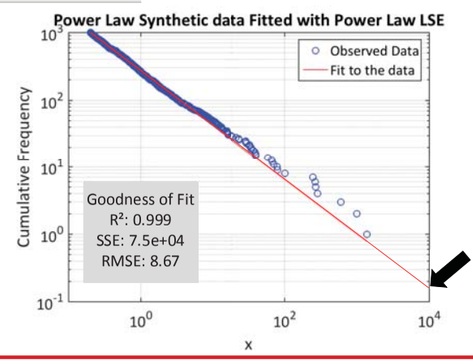
## 2. Methodology

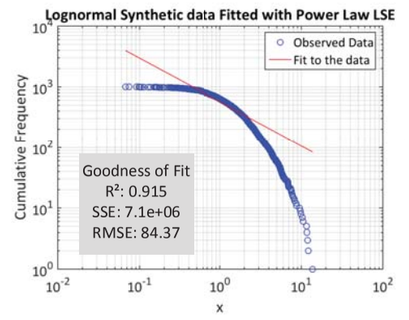
The success of any predictive model is largely dependent on the accurcy with which its parameters are known. Until now, data fitting has relied on graphical tools or LSE, rather than formal mathematical methods, yielding non-accurate paremeter estimations. **Maximum Likelihood Estimators (MLEs)** are more powerful and reliable tools, because they do not suffer subjective biases.

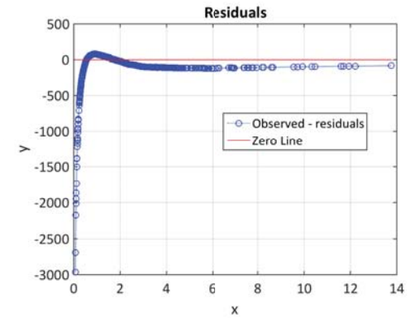
| Method | Scaling Parameter | Minimum Value |
|---|---|---|
| Known Parameter | $\alpha = 2$ | $x_{min} = 1.008$ |
| MLE | $\alpha_{MLE} = 1.991$ | $x_{min} = 1.000$ |
| LSE | $\alpha_{LSE} = 0.9972$ | |




**SYNTHETIC DATA SET**



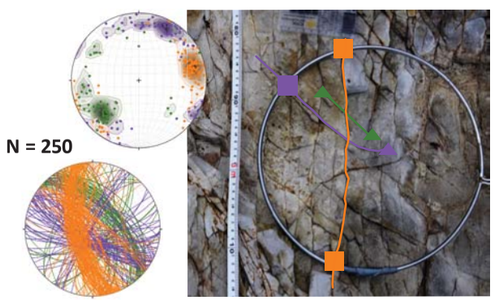




Common errors when looking for the correct distribution:
- The use of the residual plots to check the quality of the fit is commonly missed;
- Plotting e.g. a lognormal distribution on a log-log plot can mislead our interpretations and the date can be seen as power-lawed, although with many outliers

## 3. Fracture Attribute Parameters

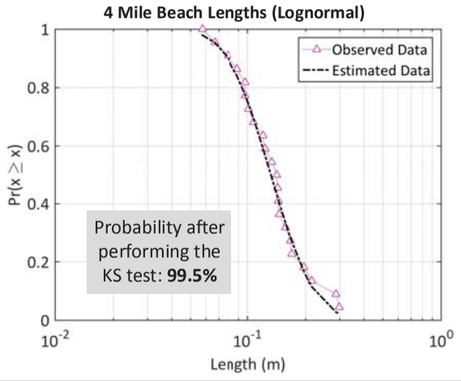
***DATA COLLECTION.*** Permeability is related to the fracture network through a **tensor** (*Oda*, 1985) and it depends on the **statistical distribution** of the fracture **Length, Aperture, Orientation**, and **Density.**

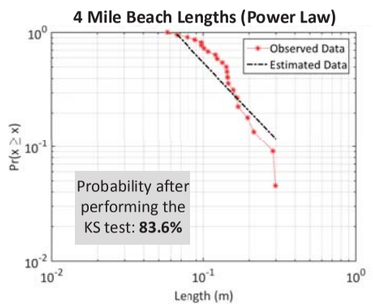
Those parameters are collected using the "Circular Estimator" (*Mauldon et al.*, 2001), which gives measuremets of the fracture mean trace lengths, density and intensity for each cirlcel along a scan-line. Apertures and orientations of each fracture inside the circle have also been collected.

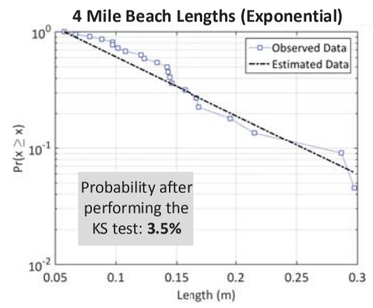



Example of the Circular Window method applied to collect fracture attributes. Squares indicate the 'n-points' (where fractures cross the edges of the window), while triangles indicate the 'm-points' (where fractures terminate inside the window).

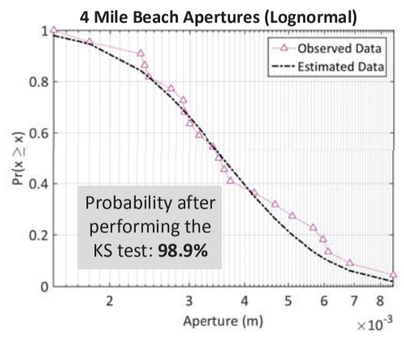
***REFERENCES.*** **Oda M.** (1985), Géotechnique; **Suzuki et al.** (1998), Inter. J. Rock Mech. Min. Sci.; **Mauldon et al.** (2001), J. Struct. Geol.; **Boehm & Moore** (2002), Geofluids; **Manzocchi T.** (2002), Water Resour. Res




***FRACTURE LENGTHS.*** To determine the best fitting curve, the data set has been tested on three of the most commonly obseved distribution for fracture attributes: power law, lognormal, exponential. With a probability of **99.5%** the **lognormal distribution** is the best distribution for fracture **trace lengths**. The computed MLE **scaling parameters** for it are $\mu_{MLE} = 2.01$ and $\sigma_{MLE} = 0.412$

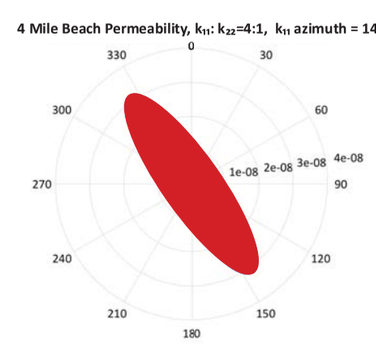



***FRACTURE APERTURES.*** As for the trace lengths, from the statistical analysis using MLE, the most likely statistic governing the distribution of fracture apertures is the **lognormal distribution**, **98.9%**. The computed MLE **scaling parameters** for it are $\mu_{MLE} = 2.01$ and $\sigma_{MLE} = 0.412$

***CORRECT SCALING PARAMETERS MEAN A BETTER PERMEABILITY ESTIMATION***

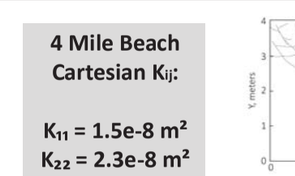
## 4. Permeability Estimations

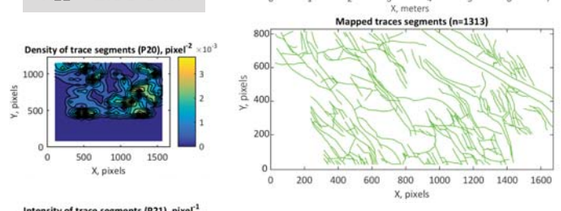
***PERMEABILITY.*** The geometry of the fractures has been simplified according to the **parallel plate model**. Following this model the **flow rate** along a fracture is **related to the cube of its aperture**. It also assumes that fractures are hosted by an impermeable medium. These assumptions allowed us to use **Oda's tensorial approach to estimate permeability** (revisited by *Suzuki et al.* (1998) for the discrete case).

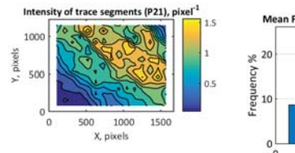


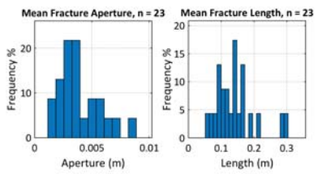

**4 Mile Beach Cartesian $K_{ij}$:**

$K_{11} = 1.5e\text{-}8\ m^2$
$K_{22} = 2.3e\text{-}8\ m^2$

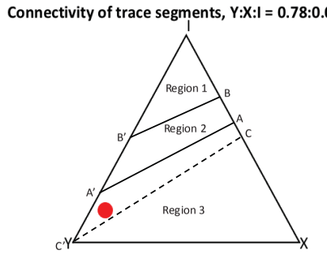



Connectivity of trace segments, Y:X:I = 0.78:0.07:0.15



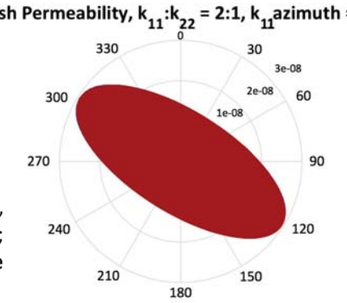

***CONNECTIVITY***. The connectivity of fracture system has been verified using **FracPaQ** (David Healy). In this plot 'I' stays for isolated fractures, 'Y' for fractures which bifurcates, and 'X' for cross-cutting fractures. Since the red dot falls in 'Region 3' (region where connectivity is maximised), **good connection between fractures is ensured** (*Manzocchi*, 2002).

## 5. Upscaling the Data

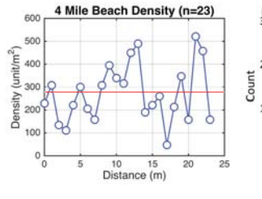
Using the correct statistical distribution of fracture attributes it is possible to create a synthetic map of the rock mass, containing a fracture network with the same statistical parameters of those measured on outcrop (***FRACTURE MESH***).

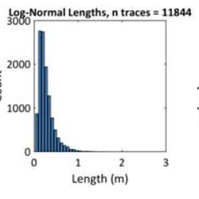
**Mesh Cartesian $K_{ij}$:**

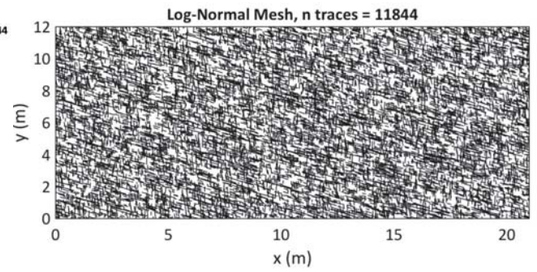
$K_{11} = 2.2e\text{-}8\ m^2$
$K_{22} = 1.8e\text{-}8\ m^2$



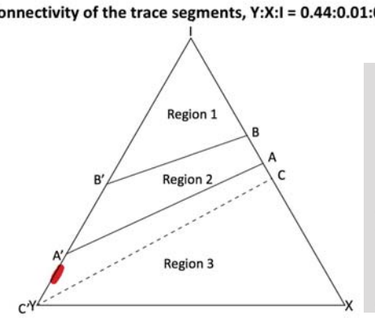

A fracture mesh has the advantage, compared with natural maps, that it is possible to configure it with variable size of resolution; moreover **it allows numerical simulations for fracture conductivity**.




Connectivity of the trace segments, Y:X:I = 0.44:0.01:0.11




- ***Fracture Meshes*** are still ***limited*** to the case of ***constant density*** of fracture distribution.

- Fracture Meshes returned ***very satisfactory permeability and connectivity values*** compared with those estimated for the natural fracture network. This will provide an ***effective method for*** studying the ***permeability*** characteristics.